The Gravitational Torsion Pendulum

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Abstract

The aim to calculate the gravitational constant via the full deflection method was successful and the value achieved, $9.6484x10^{-11}\pm 2.47489x10^{-9}10^{-9}N.m^{2}/kg^{2}$ was within the error range of the accepted value. However the calculation of the gravitational constant via the acceleration method was unsuccessful as the value for the acceleration was not attained.

Introduction and Theory

The purpose of this experiment is to measure the gravitational constant, G, using the torsion pendulum. Due to the weak nature of the gravitational force, G is only known to 3 decimal places.

The torsion balance consists of a light rod suspended by a thin bronze wire. At each end of the rod there lies a small lead ball, each of mass m. The rod is of length 2d. Two much larger lead balls, each of mass M are free to be rotated around the axis of the bronze wire on the same plane as the small balls. The angular position of the small lead balls in measured by an infrared beam which is reflected from a concave mirror attached to the centre of the rod to an array of phototransistors. The lead balls are attracted to two much larger lead spheres of mass M.

When disturbed from equilibrium the small balls undergo damped simple harmonic motion with frequency determined by the moment of inertia, I, of the small balls and the torsion constant of the wire. Moving the large balls will change the gravitational forces on the mall balls which will in turn change the angular motion of the small balls. This can be used to measure the gravitational constant. The gravitational constant can be measured by two methods; the full deflection method and the acceleration method.

## The full deflection method

When the two lead spheres are placed in position I, the small lead balls oscillate about an equilibrium position a­1. When the spheres are rotated to position II, the small lead balls will oscillate about a particular equilibrium position a­­2­. By measuring the period of the small leads balls’ oscillation, T and S, the separation of the two reflected infrared beams at angles a­1 and a­­2­, the gravitational constant is given by the following equation:

$$G=\frac{\left(2\right)\left(π^{2}\right)\left(b^{2}\right)\left(d\right)(S)}{\left(M\right)\left(T^{2}\right)(L)}$$

Where b is the distance between the centres of mass, d is the distance of the midpoints of the small lead balls from the axis, M is the mass of one of the large lead spheres and L is the distance between the mirror of the gravitational balance and the infrared position detector.

## The acceleration Method

After the spheres have been moved from to either position I to II or II to I, the balls will undergo damped oscillation. When a number of full wavelengths have been observed, the large spheres can be gently and quickly moved to the other position. The acceleration of the reflected beam, $a\_{0}^{\*}$is related to the acceleration of the small lead balls,$a\_{0}$, according to the equation:

$$a\_{0}^{\*}=a\_{0}(\frac{2L}{d})$$

Using this relation, the gravitational constant can be attained from the equation:

$$G=\frac{(a\_{0})\left(b^{2}\right)}{2M}$$

## Experimental Method

The torsion pendulum was unlocked and the large lead spheres were rotated to one of the equilibrium positions. The “CASSY lab” software was configured for the experiment to record the parameters time, t , and position, s. The measuring interval was set to 1s. The pendulum was allowed to settle into its equilibrium position and over a period of 10 minutes the displacement was monitored to ensure there were no zero-point fluctuations. Recording of the displacement with time was begun and the large lead spheres were then gently and quickly moved to the other position. Approximately 2 and a half oscillation were recorded. While the pendulum was still undergoing the oscillatory motion and being recorded by the software, the larger lead spheres were moved to their other position. The software then recorded the motion of the pendulum as it rapidly moved to perform oscillatory motion about the other equilibrium position. The velocity of this motion was plotted against the time taken and the derivative of the curve, the acceleration of the motion, was found.

When the pendulum began oscillating about its new equilibrium position, it was allowed to undergo the motion for approximately 2 and a half oscillations.

### Results and analysis

The data set for the duration of the oscillations and for the acceleration were interpreted by origin. The data corresponding to the oscillations about the equilibrium positions was interpreted and an equation of motion was applied to the oscillation. From this equation, the period time of the motion was given as $1350.36103\pm 12.82538s$. b, the distance between the centres of mass= 0.047m.d, the distance between the midpoints of the lead balls from the axis=0.05m.M, the mass of one big lead sphere=1.5kg$\pm $0.005kg. L, the distance between the mirror of the gravitational balance and the infrared detector= .65m ± .05m. S was found to be 0.0201m ±0.0001m.hence G was calculated:

$G=\frac{\left(2\right)\left(π^{2}\right)\left(0.047^{2}\right)\left(0.05\right)(0.0201)}{\left(1.5\right)\left(683^{2}\right)(0.65)}=9.6484x10^{-11}$ N.m2/kg2

The error in this value was calculated and found to be:

$$∆G=G\sqrt{\left(\frac{2∆b}{b}\right)^{2}+\left(\frac{∆d}{d}\right)^{2}+\left(\frac{∆S}{S}\right)^{2}+\left(\frac{2∆T}{1}\right)^{2}+\left(\frac{∆M}{M}\right)^{2}+\left(\frac{∆L}{L}\right)^{2}}$$

$$=9.6484x10^{-11}\sqrt{(\frac{0.0001}{0.0201})^{2}+(\frac{2(12.82538)}{1})^{2}+(\frac{0.005}{1.5})^{2}+(\frac{.05}{0.65})^{2}}=2.47489x10^{-9}N.m^{2}/kg^{2}$$

### Conclusions

The gravitational constant was calculated and found to be $9.6484x10^{-11}\pm 2.47489x10^{-9}10^{-9}N.m^{2}/kg^{2}$. The value attained was within the error calculated of the accepted value for the gravitational constant $6.67384x1o^{-11}N.m^{2}/kg^{2}$.